1. $f(t) = \int\limits_{2}^{t^2} (x+1) \, dx$

$f'(t) = 2t(t^2 + 1) + 0(2+1)$

$f'(2) = 2 \cdot 2(2^2 + 1) = 20$

Cevap: E

2. $\int\limits_{-a}^{a} (2x - 1) \, dx = -2$

$(x^2 - x)\Big|_{-a}^{a} = a^2 - a - \left((-a)^2 + a\right) = -2$

$\quad = a^2 - a - a^2 - a = -2$

$\quad = -2a = -2$

$\boxed{a = 1}$

$\int\limits_{a}^{3} (2x - 1) = (x^2 - x)\Big|_{a}^{3} = (3^2 - 3) - (a^2 - a)$

$\quad = 9 - 3 - a^2 + a$

$\quad = 9 - 3 - 1 + 1 = 6$

Cevap: E

3. $\int\limits_{0}^{1} e^{x^2} \cdot x^3 \, dx$

$x^2 = u \implies 2x \cdot dx = du$

$\qquad\qquad x \, dx = \dfrac{du}{2}$

$\int e^u \cdot u \cdot \dfrac{du}{2} = \dfrac{1}{2} \int e^u \cdot u \, du$

Türev	İntegral
U $\quad +$	e^u
1 $\quad -$	e^u
0	e^u

$\dfrac{1}{2} \cdot (u.e^u - e^u) = \dfrac{1}{2} \cdot (x^2.e^{x^2} - e^{x^2})\Big|_{0}^{1}$

$\dfrac{1}{2}(e - e) - \dfrac{1}{2}(0 - e^0) = \dfrac{1}{2}$

Cevap: C

4. $\dfrac{dy}{dx}\bigg|_{\substack{x=2 \\ y=2}} = -\dfrac{Fx}{Fy} = -\dfrac{\dfrac{y}{2\sqrt{xy}}}{\dfrac{x}{2\sqrt{xy}}}$

$\qquad = -\dfrac{\dfrac{2}{2\sqrt{4}}}{\dfrac{2}{2\sqrt{4}}} = -\dfrac{\dfrac{1}{2}}{\dfrac{1}{2}} = -1$

Cevap: B

5. $\int \dfrac{dx}{9 + 4x^2} = \int \dfrac{dx}{1 + \left(\dfrac{2x}{3}\right)^2} =$

$\dfrac{2x}{3} = u \implies \dfrac{2}{3} dx = du \implies dx = \dfrac{3}{2} du$

$\qquad\qquad = \dfrac{3}{2} \int \dfrac{1}{1 + u^2} du = \dfrac{3}{2} \arctan u + c$

$\qquad\qquad = \dfrac{3}{2} \arctan\left(\dfrac{2x}{3}\right) + c$

Cevap: C

6. $\int\limits_{x=0}^{4} \left(\int\limits_{y=0}^{3} x \, dy \right) dx = \int\limits_{x=0}^{4} \left(xy \Big|_{0}^{3} \right) dx$

$\int\limits_{x=0}^{4} (3x) \, dx = \dfrac{3x^2}{2} \Big|_{0}^{4} = \dfrac{3 \cdot 4^2}{2} = 24$

Cevap: D

7. $\int \dfrac{3x^2}{(x^3 + 1)^5} dx$

$x^3 + 1 = u \implies 3x^2 dx = du$

$\int \dfrac{1}{u^5} du = \int u^{-5} \cdot du = \dfrac{u^{-4}}{-4} + c$

$-\dfrac{1}{4} \dfrac{1}{u^4} + c = -\dfrac{1}{4} \dfrac{1}{(x^3 + 1)^4} + c$

Cevap: A

8. $\int_{0}^{3} \dfrac{x}{1+\sqrt{1+x}}\,dx$

$1 + x = u^2$

$dx = 2u\,du$

$\int \dfrac{u^2-1}{1+\sqrt{u^2}}\,2u\,du = \int \dfrac{(u-1)(u+1)}{1+u}\cdot 2u\,du$

$\int (u-1)\cdot 2u\,du = \int (2u^2 - 2u)\,du$

$\dfrac{2u^3}{3} - \dfrac{2u^2}{2} = \left(\dfrac{2(\sqrt{1+x})^3}{3} - \dfrac{2(1+x)}{2} \right) \Big|_{0}^{3}$

$\dfrac{2\cdot\sqrt{4}^{3}}{3} - \dfrac{2\cdot4}{2} - \dfrac{2\sqrt{1}}{3} + \dfrac{2}{2}$

$\dfrac{2\cdot2^3}{3} - 4 - \dfrac{2}{3} + 1 = \dfrac{14}{3} - 3 = \dfrac{5}{3}$

Cevap: E

9. $\int \dfrac{2x-3}{x^2-x}\,dx = ?$

$\dfrac{2x-3}{x(x-1)} = \dfrac{A}{x}_{(x-1)} + \dfrac{B}{x-1}_{(x)}$

$\dfrac{2x-3}{x(x-1)} = \dfrac{A(x-1)+B\cdot x}{x(x-1)}$

$2x - 3 = A(x-1) + Bx$

$x = 1$ için $2 - 3 = B$

$\boxed{B = -1}$

$x = 0$ için $-3 = -A$

$\boxed{A = 3}$

$\int \dfrac{2x-3}{x^2-x}\,dx = \int \dfrac{3}{x}\,dx - \int \dfrac{1}{x-1}\,dx$

$3\ln|x| - \ln|x-1| + c = \ln\left| \dfrac{x^3}{x-1} \right| + c$

Cevap: A

10. $\int_{0}^{a} (3x^2+1)\,dx = \int_{0}^{a} (2x+3)\,dx$

$\left(\dfrac{3x^3}{3} + x \right)\Big|_{0}^{a} = \left(\dfrac{2x^2}{2} + 3x \right)\Big|_{0}^{a}$

$a^3 + a - 0 = a^2 + 3a - 0$

$a^3 - a^2 + a - 3a = 0$

$a^3 - a^2 - 2a = 0$

$a(a^2 - a - 2) = 0$

$a = 2$

Cevap: B

11. $\int_{1}^{3} f(x)\,dx = \int_{1}^{e} f(x)\,dx + \int_{e}^{3} f(x)\,dx$

$= \int_{1}^{e} (x-1)\,dx + \int_{e}^{3} x\,dx$

$= \left(\dfrac{x^2}{2} - x \right)\Big|_{1}^{e} + \dfrac{x^2}{2}\Big|_{e}^{3}$

$= \dfrac{e^2}{2} - e - \dfrac{1}{2} + 1 + \dfrac{3^2}{2} - \dfrac{e^2}{2}$

$= -e - \dfrac{1}{2} + 1 + \dfrac{9}{2}$

$= -e + 4 + 1 = -e + 5$

Cevap: B

12. $f(x) = \dfrac{x^3}{3} - \dfrac{2x^2}{2} + x + c$

$f(3) = \dfrac{27}{3} - 9 + 3 + c = 5$

$c = 2$

$f(0) = 0 - 0 + 0 + 2 = 2$

Cevap: A

13. $\int x^{\frac{1}{\ln x}} dx = \int x^{\log_x e} dx = \int e^{\log_x x} dx$

$= \int e\,dx = ex \Big|_2^3$

$= 3e - 2e = e$

Cevap: D

16. $\int_1^{e^2} \frac{\ln^2 x}{2x} dx$

$\ln x = u \implies \frac{1}{x} dx = du$

$x = 1$ için $u = 0$

$x = e^2$ için $u = 2$

$\int_0^2 \frac{u^2}{2} \cdot du = \frac{u^3}{6} \Big|_0^2 = \frac{2^3}{6} = \frac{4}{3}$

Cevap: A

14. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^3 x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2 x.\cos x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 - \sin^2 x).\cos x\,dx$

$\sin x = u \implies \cos x.dx = du$

$= \int (1 - u^2).du = u - \frac{u^3}{3} = \left(\sin x - \frac{\sin^3 x}{3} \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\sin\frac{\pi}{3} - \frac{\sin^3\frac{\pi}{3}}{3} - \sin\frac{\pi}{6} + \frac{\sin^3\frac{\pi}{6}}{3} =$

$\frac{\sqrt{3}}{2} - \frac{3\sqrt{3}}{24} - \frac{1}{2} - \frac{1}{24} =$

$\underset{(12)}{\frac{\sqrt{3}}{2}} - \underset{(3)}{\frac{\sqrt{3}}{8}} - \underset{(12)}{\frac{1}{2}} + \frac{1}{24} = \frac{12\sqrt{3} - 3\sqrt{3} - 12 + 1}{24}$

$= \frac{9\sqrt{3} - 11}{24}$

Cevap: C

17. $\frac{1}{(x - 2)(x + 2)} = \frac{A}{\underset{(x+2)}{x - 2}} + \frac{B}{\underset{(x-2)}{x + 2}}$

$1 = A(x + 2) + B(x - 2)$

$x = -2 \qquad 1 = -4B \qquad B = -\frac{1}{4}$

$x = 2 \qquad 1 = 4A \qquad A = \frac{1}{4}$

$4\int \left(\frac{\frac{1}{4}}{x - 2} + \frac{-\frac{1}{4}}{x + 2} \right) dx =$

$\int \left(\frac{1}{x - 2} - \frac{1}{x + 2} \right) dx = \ln|x - 2| - \ln|x + 2| \Big|_0^1$

$= \ln\left| \frac{x - 2}{x + 2} \right| \Big|_0^1$

$= \ln\frac{1}{3} - \ln 1$

$= -\ln 3$

Cevap: B

15. $-\frac{Fx}{Fy} = -\frac{4x - 1}{-2y} = \frac{4x - 1}{2y} \Big|_{\substack{x = 1 \\ y = 1}}$

$= \frac{4.1 - 1}{2.1} = \frac{3}{2}$

Cevap: B

18. $\int \frac{x^3 + 3x^2 + 1}{x} dx = \int \left(x^2 + 3x + \frac{1}{x} \right) dx$

$= \frac{x^3}{3} + \frac{3x^2}{2} + \ln|x| + c$

Cevap: B

TASARI EĞİTİM YAYINLARI

19. $\int\limits_{1}^{e} x\ln x\,dx + \int\limits_{1}^{e} x\ln^2 x\,dx$

$\ln^2 x = u \quad \rightarrow \quad 2\ln x \cdot \dfrac{1}{x}\,dx = du$

$x\,dx = dv \quad \rightarrow \quad \dfrac{x^2}{2} = v$

$\int x\ln x\,dx + \int x\ln^2 x\,dx = \ln^2 x \cdot \dfrac{x^2}{2} - \int \dfrac{x^2}{2} \cdot 2.\ln x \cdot \dfrac{1}{x}\,dx$

$\qquad\qquad = \ln^2 x \cdot \dfrac{x^2}{2} - \int x\ln x\,dx + \int x\ln x\,dx$

$\qquad\qquad = \ln^2 x \cdot \dfrac{x^2}{2}\Big|_{1}^{e} = \ln^2 e \cdot \dfrac{e^2}{2} - 0$

$\qquad\qquad = \dfrac{e^2}{2}$

Cevap: C

20. $\int\limits_{e}^{e^2} \dfrac{dx}{x\ln x} =$

$\ln x = u \quad \Rightarrow \quad \dfrac{1}{x}\,dx = du$

$\int\limits_{1}^{2} \dfrac{1}{u}\,du = \ln|u|\Big|_{1}^{2} = \ln|2| - \ln|1|$

$\qquad\qquad = \ln 2$

Cevap: B