

1. $A^T = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$

$$A^T + A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Cevap: A

2. $A \cdot A^{-1} = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3a - 2b & -a + 2b \\ 3c - 2d & -c + 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 3a - 2b = 1 \\ -a + 2b = 0 \end{cases} \Rightarrow \begin{cases} 3a - 2b = 1 \\ a = 2b \end{cases} \Rightarrow \begin{cases} 3(2b) - 2b = 1 \\ 4b = 1 \end{cases} \Rightarrow \begin{cases} 6b - 2b = 1 \\ b = \frac{1}{4} \end{cases}$$

$$a = 2b = 2 \cdot \frac{1}{4} = \frac{1}{2}$$

$$3c - 2d = 0 \Rightarrow 3c = 2d \Rightarrow d = \frac{3c}{2}$$

$$-c + 2d = 1 \Rightarrow -c + 2 \cdot \frac{3c}{2} = 2c = 1 \Rightarrow c = \frac{1}{2}$$

$$d = \frac{3 \cdot \frac{1}{2}}{2} = \frac{3}{4}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

Cevap: B

3. $A = \begin{bmatrix} 1 & x \\ 1 & y \end{bmatrix}, B = \begin{bmatrix} y & 1 \\ x & 1 \end{bmatrix}$

$$|A \cdot B| = \left| \begin{bmatrix} 1 & x \\ 1 & y \end{bmatrix} \cdot \begin{bmatrix} y & 1 \\ x & 1 \end{bmatrix} \right|$$

$$= \begin{vmatrix} y + x^2 & 1 + x \\ y + xy & 1 + y \end{vmatrix}$$

$$= (y + x^2) \cdot (1 + y) - (y + xy)(1 + x)$$

$$= y + y^2 + x^2 + x^2y - y - yx - xy - x^2y$$

$$= x^2 - 2xy + y^2 = (x - y)^2$$

Cevap: B

4.

$$\begin{vmatrix} 0 & k & 3 \\ -2 & -1 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & k \\ -10k & -2 & -1 \end{vmatrix} \begin{vmatrix} 3 \\ 5 \\ 0 \\ 3 \\ 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ -18 \\ 0 \\ 0 \end{vmatrix}$$

$$-18 - (-10k) = 2$$

$$-20 = -10k$$

$$\boxed{2 = k}$$

Cevap: B

5. $A \cdot B = C$

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ olsun.}$$

$$\begin{bmatrix} 6 & 3 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 6a + 3c & 6b + 3d \\ 0 + 6c & 0 + 6d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}$$

$$6c = 0 \Rightarrow c = 0$$

$$6d = 8 \Rightarrow d = \frac{4}{3}$$

$$6a + 0 = 2 \text{ ve } 6b + \frac{3 \cdot 4}{3} = 1$$

$$a = \frac{1}{3}$$

$$6b = -3$$

$$b = -\frac{1}{2}$$

$$B = \begin{bmatrix} \frac{1}{3} & -\frac{1}{2} \\ 0 & \frac{4}{3} \end{bmatrix}$$

Cevap: D

6. $A - 3 \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix} + 6 = 0$

$$A = \begin{bmatrix} 3 & 15 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 15 \\ 6 & 6 \end{bmatrix}$$

Cevap: A

7. $\begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 5 \end{bmatrix} =$

$$\begin{bmatrix} -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -3 \end{bmatrix}$$

Cevap: D

8. $x_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\begin{cases} x_1 + x_2 = a \\ x_1 - 2x_2 = b \end{cases} \quad \begin{matrix} 2x_1 + 2x_2 = 2a \\ + \quad x_1 - 2x_2 = b \end{matrix}$$

$$3x_1 = 2a + b$$

$$x_1 = \frac{2a + b}{3}$$

Cevap: C

9. $f(A) = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}^2 + 3 \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 \\ 6 & -4 \end{bmatrix}$$

Cevap: B

10. $A^2 = \begin{bmatrix} -2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} = 16 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -32 & 48 \\ 48 & 32 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} -32 & 48 \\ 48 & 32 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 4 & 2 \end{bmatrix} = 16 \begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\vdots = 16 \cdot 16 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2^8 \cdot I$$

$$A^{2006} = 2^{4012} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Cevap: A

11. $A + B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \quad \left\{ \begin{array}{l} 2A + 2B = \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} \\ A - 2B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \end{array} \right.$

$$+ \quad A - 2B = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$3A = \begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{5}{3} & \frac{7}{3} \\ 1 & \frac{4}{3} \end{bmatrix}$$

Cevap: A

12. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -7 & -8 & -9 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = -A + A = 0$

Cevap: A

13. $\begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix}^2 = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} \begin{bmatrix} x & 2 \\ y & -1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$

$$\begin{bmatrix} x^2 + 2y & 2x - 2 \\ xy - y & 2y + 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

$$2x - 2 = 2 \Rightarrow \boxed{x = 2}$$

$$2y + 1 = 3 \Rightarrow \boxed{y = 1}$$

$(x, y) = (2, 1)$ olur.

Cevap: D

14. $a = 1997$ olsun.

$$\begin{bmatrix} 1996 & 1998 \\ 1997 & 1999 \end{bmatrix} = \begin{bmatrix} a-1 & a+1 \\ a & a+2 \end{bmatrix}$$

$$\begin{aligned} \det(A) &= (a-1)(a+2) - a(a+1) \\ &= a^2 + 2a - a - 2 - a^2 - a \\ &= -2 \end{aligned}$$

Cevap: D

15. $A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{1.4 - (-2).3} \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$$

veya $= \frac{-1}{10} \begin{bmatrix} -4 & -2 \\ 3 & -1 \end{bmatrix}$

Cevap: B

16. $x \begin{bmatrix} 2 \\ 1 \end{bmatrix} - y \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -13 \end{bmatrix}$

$$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3y \\ -4y \end{bmatrix} = \begin{bmatrix} 7 \\ -13 \end{bmatrix}$$

$$\begin{array}{rcl} \left. \begin{array}{l} 2x + 3y = 7 \\ -2x - 4y = -13 \end{array} \right\} & + & \begin{array}{l} 2x + 3y = 7 \\ -2x + 8y = 26 \end{array} \\ & & \hline & & 11y = 33 \\ & & \boxed{y = 3} \end{array}$$

$$2x + 3y = 7 \Rightarrow 2x + 3.3 = 7$$

$$2x = -2 \quad \boxed{x = -1}$$

$$y - x = 3 - (-1) = 3 + 1 = 4 \text{ bulunur.}$$

Cevap: A

17. $B^{-1} = \frac{1}{2.2 - 1.3} \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$$A.B^{-1} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -6 & 4 \end{bmatrix}$$

Cevap: A

18. $A + B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ -2 & 0 \end{bmatrix}$

$$A + B = \begin{bmatrix} 0 & 3 \\ -2 & 3 \end{bmatrix}$$

$$\det(A + B) = 0.3 - (-2).3 = 6$$

Cevap: D

19. $A + 2B = \begin{bmatrix} 3 & -4 \\ 0 & 6 \end{bmatrix} \Rightarrow 3A + 6B = \begin{bmatrix} 9 & -12 \\ 0 & 18 \end{bmatrix}$

$$A - 3B = \begin{bmatrix} 3 & 1 \\ 5 & -4 \end{bmatrix} \Rightarrow + 2A - 6B = \begin{bmatrix} 6 & 2 \\ 10 & -8 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & -10 \\ 10 & 10 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix}$$

Cevap: A

20. $|A| = a.d - b.c = 5$

$$|B| = 4d(3a - b) - 4b(3c - d)$$

$$= 12ad - 4db - 12bc + 4bd$$

$$= 12ad - 12bc = 12(ad - bc)$$

$$= 12.5 = 60$$

Cevap: E