

1. $x \neq 0, y \neq 0$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{4}$$

$$\frac{1}{x^2} - \frac{1}{y^2} = \frac{3}{16} \Rightarrow \underbrace{\left(\frac{1}{x} - \frac{1}{y}\right)}_{\frac{1}{4}} \cdot \left(\frac{1}{x} + \frac{1}{y}\right) = \frac{3}{16}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{3}{16} \cdot 4$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{3}{4}$$

$$+ \frac{1}{x} - \frac{1}{y} = \frac{1}{4}$$

$$\underline{\underline{\frac{2}{x} = 1 \rightarrow x = 2}}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{3}{4} \Rightarrow \frac{1}{2} + \frac{1}{y} = \frac{3}{4}$$

$$\frac{1}{y} = \frac{3}{4} - \frac{1}{2} \quad (1) \quad (2)$$

$$\frac{1}{y} = \frac{1}{4}$$

$$\Rightarrow y = 4$$

$$x + y = 2 + 4 = 6$$

Cevap: C

2. $a.b = 3$

$$\frac{1}{a^2} + \frac{1}{b^2} = 2 \Rightarrow \frac{a^2 + b^2}{(a.b)^2} = 2$$

$$\Rightarrow \frac{a^2 + b^2}{3^2} = 2$$

$$\Rightarrow a^2 + b^2 = 18$$

$$\Rightarrow (a - b)^2 = a^2 - 2a.b + b^2 = \underbrace{a^2 + b^2}_{18} - \underbrace{2a.b}_{2 \cdot 3} = 18 - 6 = 12$$

Cevap: C

3. $\frac{2x^2}{x} - \frac{5x}{x} + \frac{2}{x} = \frac{0}{x} \rightarrow$ Verilen denklemin bütün terimlerini x 'e bölelim.

$$2x - 5 + \frac{2}{x} = 0 \Rightarrow 2x + \frac{2}{x} = 5$$

$$\left(2x + \frac{2}{x}\right)^2 = 5^2$$

$$4x^2 + \frac{4}{x^2} + 2 \cdot 2x \cdot \frac{2}{x} = 25$$

$$4x^2 + \frac{4}{x^2} = 25 - 8 = 17$$

Cevap: A

4. $\frac{x^2 - 2}{x - \sqrt{2}} + 2 - \sqrt{2} = 5$

$$\Rightarrow \frac{(x - \sqrt{2})(x + \sqrt{2})}{x - \sqrt{2}} + 2 - \sqrt{2} = 5$$

$$\Rightarrow x + \sqrt{2} + 2 - \sqrt{2} = 5$$

$$\Rightarrow x + 2 = 5$$

$$\Rightarrow x = 3$$

Cevap: C

5. $\frac{x^3 + y^3}{(x + y)^2} \cdot \frac{x + y}{xy - x^2 - y^2}$

$$= \frac{(x+y)(x^2 - xy + y^2)}{(x+y)^2} \cdot \frac{x+y}{-(x^2 - xy + y^2)} = -1$$

Cevap: A

6. $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

Binom açılımından:

$$\begin{aligned} \left(\frac{x}{y} - \frac{1}{x}\right)^4 &= \left(\frac{x}{y}\right)^4 - 4 \cdot \left(\frac{x}{y}\right)^3 \cdot \frac{1}{x} + 6 \cdot \left(\frac{x}{y}\right)^2 \cdot \left(\frac{1}{x}\right)^2 - 4 \cdot \left(\frac{x}{y}\right) \cdot \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^4 \\ &= \left(\frac{x}{y}\right)^4 - 4 \cdot \left(\frac{x}{y}\right)^3 \cdot \frac{1}{x} + 6 \cdot \left(\frac{x}{y}\right)^2 \cdot \left(\frac{1}{x}\right)^2 - 4 \cdot \left(\frac{x}{y}\right) \cdot \left(\frac{1}{x}\right)^3 + \left(\frac{1}{x}\right)^4 \end{aligned}$$

Terimler çarpımı:

$$\frac{x^4}{y^4} \cdot -4 \cdot \frac{x^3}{y^3} \cdot \frac{1}{x} \cdot 6 \cdot \frac{x^2}{y^2} \cdot \frac{1}{x^2} \cdot -4 \cdot \frac{x}{y} \cdot \frac{1}{x^3} \cdot \frac{1}{x^4} = \frac{96}{y^{10}}$$

Cevap: A

9. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ açılımından;

$$\begin{aligned} \frac{79^3 - 21^3}{79 \cdot 10^2 + 21^2} &= \frac{(79 - 21)(79^2 + 79 \cdot 21 + 21^2)}{79 \cdot 10^2 + 21^2} \\ &= \frac{58 \cdot (79 + 21) + 21^2}{79 \cdot 10^2 + 21^2} \\ &= \frac{58 \cdot (79 + 21) + 21^2}{79 \cdot 10^2 + 21^2} \\ &= 58 \end{aligned}$$

Cevap: B

7. $x + \sqrt{x} - \sqrt[3]{x} = 68 \Rightarrow x = ?$

Bu tür bir soruda, şıkları incelersek; C) şıklındaki 64 sayısını x yerine yazdığımızda eşitliğin sağlandığını görürüz.

$$\begin{aligned} [x = 64] \text{ için } x + \sqrt{x} - \sqrt[3]{x} &= 64 + \sqrt{64} - \sqrt[3]{64} \\ &= 64 + 8 - 4 \\ &= 68 \end{aligned}$$

Cevap: C

8. $x, y \in \mathbb{Z}^+$

$$x^2 + 6x - 6y - y^2 = 20 \Rightarrow x^2 - y^2 + 6x - 6y = 20$$

$$x + y = 4 \quad (x - y)(x + y) + 6(x - y) = 20$$

$$x^2 - y^2 = ? \quad (x - y) \cdot \underbrace{(x + y)}_4 = 20$$

$$(x - y) \cdot 10 = 20$$

$$x - y = 2$$

$$x^2 - y^2 = (x - y)(x + y) = 2 \cdot 4 = 8$$

Cevap: A

10. $x > y$ $\left. \begin{array}{l} x^2 + y^2 = \sqrt{5} \\ x \cdot y = 1 \end{array} \right\} x^4 - y^4 = ?$

$$(x^2 + y^2)^2 = (\sqrt{5})^2 \Rightarrow x^4 + 2x^2 \cdot y^2 + y^4 = 5$$

$$\underbrace{x^4 + y^4}_1 + 2 \cdot \underbrace{(x \cdot y)^2}_1 = 5$$

$$x^4 + y^4 = 3$$

$$(x^2 - y^2)^2 = x^4 - 2 \cdot x^2 \cdot y^2 + y^4 = \underbrace{x^4 + y^4}_3 - 2 \cdot \underbrace{(x \cdot y)^2}_1 = 1$$

$$\Rightarrow x^2 - y^2 = 1$$

$$\Rightarrow x^4 - y^4 = \underbrace{(x^2 - y^2)}_1 \cdot \underbrace{(x^2 + y^2)}_{\sqrt{5}} = \sqrt{5}$$

Cevap: C

11. $x^2 - y^2 = 5$
 $x \cdot y = \sqrt{17}$ $\left. \Rightarrow x^2 + y^2 = ? \right.$

$$\begin{aligned} 1) \quad (x^2 - y^2)^2 &= 5^2 \Rightarrow x^4 - 2x^2y^2 + y^4 = 25 \\ &\Rightarrow x^4 + y^4 - 2(x \cdot y)^2 = 25 \\ &\qquad\qquad\qquad \overline{\sqrt{17}} \\ &\Rightarrow x^4 + y^4 = 25 + 2 \cdot 17 = 25 + 34 = 59 \\ 2) \quad (x^2 + y^2)^2 &= x^4 + 2x^2y^2 + y^4 = \underbrace{x^4 + y^4}_{59} + 2 \cdot (x \cdot y)^2 \\ &\qquad\qquad\qquad \overline{\sqrt{17}} \\ &\Rightarrow (x^2 + y^2)^2 = 59 + 2 \cdot 17 = 59 + 34 = 93 \\ &\Rightarrow x^2 + y^2 = \sqrt{93} \end{aligned}$$

Cevap: D

$$\begin{aligned} 12. \quad &\underbrace{(2^x - 1) \cdot (2^x + 1)}_{(2^{2x} - 1)} \cdot (2^{2x} + 1) \cdot (2^{4x} + 1) = 1023 \\ &\underbrace{(2^{2x} - 1) \cdot (2^{2x} + 1)}_{(2^{4x} - 1)} \cdot (2^{4x} + 1) = 1023 \\ &\underbrace{(2^{4x} - 1) \cdot (2^{4x} + 1)}_{2^{8x} - 1} = 1023 \\ &2^{8x} - 1 = 1023 \Rightarrow 2^{8x} = 1024 \\ &\Rightarrow 2^{8x} = 2^{10} \\ &\Rightarrow 8x = 10 \\ &\Rightarrow x = \frac{10}{8} = \frac{5}{4} \end{aligned}$$

Cevap: E

$$\begin{aligned} 13. \quad a + \frac{1}{2a} &= \sqrt{10} \Rightarrow \left(a + \frac{1}{2a}\right)^2 = (\sqrt{10})^2 \\ &\Rightarrow a^2 + 2 \cdot a \cdot \frac{1}{2a} + \frac{1}{4a^2} = 10 \\ &\Rightarrow a^2 + \frac{1}{4a^2} + 1 = 10 \\ &\Rightarrow a^2 + \frac{1}{4a^2} = 9 \\ &\Rightarrow \sqrt{a^2 + \frac{1}{4a^2}} = 3 \end{aligned}$$

Cevap: B

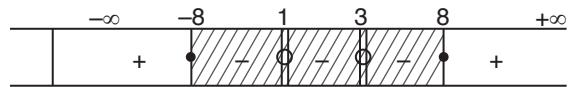
14. $\frac{(a^2 - 64)^{1991}}{(a-1)^{2000} \cdot (a-3)^{1990}} \leq 0$

$a^2 - 64 = 0 \Rightarrow a^2 = 64 \Rightarrow \boxed{a=-8}, \boxed{a=8}$

$a-1=0 \Rightarrow \boxed{a=1}, a-3=0 \Rightarrow \boxed{a=3}$

çift katlı kök

çift katlı kök



$\mathcal{C.K} = [-8, 8] - \{1, 3\}$ (1 ve 3 paydayı sıfır yaptığı için çözüm kümesine dahil edemeyiz.)

Cevap: D

$$\begin{aligned} 15. \quad 2x = y &\Rightarrow \frac{x}{y} = \frac{1}{2} \Rightarrow \frac{x}{y} = \frac{5}{10} \\ 2y = 5z &\Rightarrow \frac{y}{z} = \frac{5}{2} \Rightarrow \frac{y}{z} = \frac{10}{4} \end{aligned} \right. \Rightarrow \begin{aligned} x &= 5k \\ y &= 10k \\ z &= 4k \end{aligned}$$

$$\begin{aligned} \left(\frac{x}{z}\right)^{-\frac{y}{z}} &= \left(\frac{5k}{4k}\right)^{-\frac{10k}{4k}} = \left(\frac{5}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{5}\right)^{\frac{1}{2}} \\ &= \sqrt{\frac{4}{5}} \\ &= \frac{2}{\sqrt{5}} \\ &= \frac{2\sqrt{5}}{5} \end{aligned}$$

Cevap: C

$$\begin{aligned} 16. \quad a &= \sqrt{2} - 1 \\ a^2 + 3a - 10 &= (a-2)(a+5) = (\sqrt{2}-1-2)(\sqrt{2}-1+5) \\ \downarrow &\quad \downarrow \\ a &\quad -2 \\ a &\quad 5 \end{aligned} \Rightarrow \begin{aligned} &= (\sqrt{2}-3)(\sqrt{2}+4) \\ &= 2 + 4\sqrt{2} - 3\sqrt{2} - 12 \\ &= \sqrt{2} - 10 \end{aligned}$$

Cevap: B

17. $2017^2 - 2016^2 = (2017 - 2016).(2017 + 2016)$
 $= 1.4033$
 $= 4033$

Cevap: C

18. $\frac{2x^2+x-1}{x^2-1} : \frac{2x^2+5x-3}{x^2+2x-3}$

2x	-1	2x	-1
x	1	x	3
↑	↑	↑	↑
↓	↓	x	3
x		-1	

$$\begin{aligned} &= \frac{(2x-1).(x+1)}{(x-1).(x+1)} : \frac{(2x-1).(x+3)}{(x+3).(x-1)} \\ &= \frac{2x-1}{x-1} \cdot \frac{x-1}{2x-1} \\ &= 1 \end{aligned}$$

19. $a, b \in \mathbb{Z}; a + b = 2$

$$\begin{aligned} a^2 + 5a - b^2 + 5b &= 14 \Rightarrow a^2 - b^2 + 5a + 5b = 14 \\ &\Rightarrow (a-b).(a+b) + 5.(a+b) = 14 \\ &\Rightarrow (a+b).(a-b+5) = 14 \\ &\quad \overline{ } \\ &\Rightarrow a-b+5 = 7 \\ &\Rightarrow a-\cancel{b} = 2 \\ &\quad + a+\cancel{b} = 2 \\ &\quad \overline{2a = 4 \rightarrow a = 2} \\ &a+b = 2 \rightarrow 2+b = 2 \\ &\rightarrow \boxed{b = 0} \end{aligned}$$

Cevap: A

20. $x, y \in \mathbb{R}$

$\underbrace{2x^2 + y^2}_{x^2 + x^2} + 2xy - 4x + 4 = 0$

$x^2 + x^2 + y^2 + 2xy - 4x + 4 = 0$

$\underbrace{x^2 - 4x + 4}_{(x-2)^2} + \underbrace{x^2 + 2xy + y^2}_{(x+y)^2} = 0$

$(x-2)^2 + (x+y)^2 = 0$

$\Rightarrow x-2 = 0 \Rightarrow x = 2$

$x+y = 0 \Rightarrow 2+y = 0 \Rightarrow y = -2$

$\Rightarrow x+3y = 2+3.(-2) = 2-6 = -4$

Cevap: B